tsfilter hp uses the Hodrick–Prescott (HP) high-pass filter to separate a time-series y_t into trend and cyclical components

$$y_t = \tau_t + c_t$$

where τ_t is the trend component and c_t is the cyclical component. τ_t may be nonstationary; it may contain a deterministic or a stochastic trend, as discussed below.

The primary objective is to estimate c_t, a stationary cyclical component that is driven by stochastic cycles at a range of periods. The trend component τ_t is calculated by the difference $\tau_t = y_t - c_t$.

Formally, the filter is defined as the solution to the following optimization problem for τ_t

$$\min_{\tau_t} \left[\sum_{t=1}^{T} (y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} \{(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})\}^2 \right]$$

where the smoothing parameter λ is set fixed to a value.

If $\lambda = 0$, the solution degenerates to $\tau_t = y_t$, in which case the filter excludes all frequencies, that is, $c_t = 0$. On the other extreme, as $\lambda \to \infty$, the solution approaches the least-squares fit to the line $\tau_t = \beta_0 + \beta_1 t$; see Hodrick and Prescott (1997) for a discussion.

For a fixed λ, it can be shown that the cyclical component $\mathbf{c}' = (c_1, c_2, \ldots, c_T)$ is calculated by

$$\mathbf{c} = (\mathbf{I_T} - \mathbf{M}^{-1})\mathbf{y}$$

where $\mathbf{y}$ is the column vector $\mathbf{y}' = (y_1, y_2, \ldots, y_T)$, $\mathbf{I_T}$ is the $T \times T$ identity matrix, and $\mathbf{M}$ is the $T \times T$ matrix:

$$\mathbf{M} = \begin{pmatrix}
(1+\lambda) & -2\lambda & \lambda & 0 & 0 & 0 & \ldots & 0 \\
-2\lambda & (1+5\lambda) & -4\lambda & \lambda & 0 & 0 & \ldots & 0 \\
\lambda & -4\lambda & (1+6\lambda) & -4\lambda & \lambda & 0 & \ldots & 0 \\
0 & \lambda & -4\lambda & (1+6\lambda) & -4\lambda & \lambda & \ldots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ldots & \vdots \\
\vdots & 0 & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\
0 & \ldots & \lambda & -4\lambda & (1+6\lambda) & -4\lambda & \lambda & 0 \\
0 & \ldots & 0 & \lambda & -4\lambda & (1+6\lambda) & -4\lambda & \lambda \\
0 & \ldots & 0 & 0 & \lambda & -4\lambda & (1+5\lambda) & -2\lambda \\
0 & \ldots & 0 & 0 & 0 & \lambda & -2\lambda & (1+\lambda)
\end{pmatrix}$$

The gain of the HP filter is given by (see King and Rebelo [1993], Maravall and del Rio [2007], or Harvey and Trimbur [2008])

$$\psi(\omega) = \frac{4\lambda\{1 - \cos(\omega)\}^2}{1 + 4\lambda\{1 - \cos(\omega)\}^2}$$

As discussed in [TS] **tsfilter**, there are two approaches to selecting λ. One method, based on the heuristic argument of Hodrick and Prescott (1997), is used to compute the default values for λ. The method sets λ to 1,600 for quarterly data and to the rescaled values worked out by Ravn and Uhlig (2002). The rescaled default values for λ are 6.25 for yearly data, 100 for half-yearly data, 129,600 for monthly data, 1600×12^4 for weekly data, and $1600 \times (365/4)^4$ for daily data.
Additional technical description is available from Stata's website.